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THE APPLICATION OF THE NORTON-BAILEY LAW FOR CREEP PREDICTION THROUGH POWER LAW REGRESSION

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ABSTRACT

Accurate determination of constitutive modeling constants used in high value components, especially in electric power generation equipment, is vital for related design activities. Parts under creep are replaced after extensive deformation is reached, so models, such as the Norton-Bailey power law, support service life prediction and repair/replacement decisions. For high fidelity calculations, experimentally acquired creep data must be accurately regressed over a variety of temperature, stress, and time combinations. If these constants are not precise, then engineers could be potentially replacing components with lives that have been fractionally exhausted, or conversely, allowing components to operate that have already been exhausted. By manipulating the Norton-Bailey law and utilizing bivariate power-law statistical regression, a novel method is introduced to precisely calculate creep constants over a variety of sets of data. The limits of the approach are explored numerically and analytically.

optimize these constants typically involve manual curve-fitting to creep data in order to acquire best fits across several creep curves. If the constants found were their true values, then plotting the Norton-Bailey values versus time would result in a near-perfect match of the data. In some situations, the constant determination is hampered by sparse data sets at intervals of strain (e.g. 0.1%, 0.5%) or at constant times (1 hr, 10 hr).

Research was conducted to develop a formulation to identify power law creep constants that would result in an optimal fit with creep data across test variables of both stress and temperature. The purpose of this investigation is to develop a reliable approach to regressing multivariate power law type data. A background look at creep deformation, other creep models, and general approaches to constant determination are discussed next. Following that, the methods being investigated are derived and tested on both physical and simulated data and its limitations are discussed.

INTRODUCTION

Material selection is a critical stage in mechanical design engineering of structural components. Perhaps the most important consideration for parts subjected to long term use are expected service life, acceptable deformation rate, and the environment in which the material will be used. In order to accurately determine creep rupture life, engineers use analytical approaches to simulate the primary and secondary creep response. An example of such a model is the Norton-Bailey model, which contains three temperature dependent regression constants. The methods used to

CREEP DEFORMATION

Constitutive models have been developed to interpolate and predict the deformation behavior of materials exhibiting time-dependent, inelastic deformation. A model commonly applied for the primary and secondary creep regimes was developed by Bailey and Norton [1], i.e.,

$$\epsilon_{cr} = A\sigma^n t^m \quad (1)$$

where A , n , and m are temperature dependent material constants that are generally independent of stress. While n and m are unitless, the creep strain hardening coefficient, A , has units that are consistent with those of

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time, t , and stress, σ . The time-differentiated version of this expression is often referred to as the time-hardening formulation of power law creep. Solving the equation above for t and incorporating that into the time-differentiated formulation yields the following:

$$\dot{\epsilon}_{cr} = mA \frac{1}{m} \sigma \frac{n}{m} (\epsilon_{cr})^{\frac{m-1}{m}} \quad (2)$$

This is called the strain hardening formulation of power law creep. In practice, the time and strain hardening formulations are used to predict the creep strain histories at fixed stress and temperature levels. Experience indicates that the strain hardening formulation often produces better agreement with the results of actual tests under variable stress.

In literature, the Norton-Bailey law has been expressed in another form [1]. The rate formulation of the rule is given a modification of Eq. (1), i.e.

$$\dot{\epsilon}_{cr} = A' \sigma^{n'} t^{m'} \quad (3)$$

where A' , n' , and m' remain as temperature-dependent constants as in Eq. (1), but A' has units of $\text{MPa}^{-n'} \text{hr}^{-m'-1} \%$ or $\text{MPa}^{-n'} \text{hr}^{-m'-1}$. The Eq. (3) form of the Norton-Bailey law has been used with m' equal to zero [2]. This form also has the restrictions that A' must be greater than zero. These models are suitable when primary and secondary creep dominate the history, as seen in Fig. 1.

Another notable constitutive model that has been applied for the prediction of creep is the theta-project approach [3]. In this approach, the complete creep curve is simulated by

$$\epsilon_{cr} = \theta_1 \left[1 - e^{(-\theta_2 t)} \right] + \theta_3 \left[e^{(\theta_4 t)} - 1 \right] \quad (4)$$

Here the θ terms are regression constants that allow the formulation to interpolate the primary and tertiary regimes of creep. This is a plausible model for primary dominant creep if θ_3 is set to zero. Secondary creep is not explicitly accounted for, but is generally predicted well if the initial and final responses are closely curve fit. In separate studies, Parker (1985) and Ghosh and Chaudhuri (1994) developed modeling constants for 2.25Cr-1Mo, a low alloy steel, at 538°C at stresses ranging from 100MPa

(14.5ksi) to 300MPa (43.5ksi). This model also represents a case in which constants are determined for fixed levels of stress and temperature.

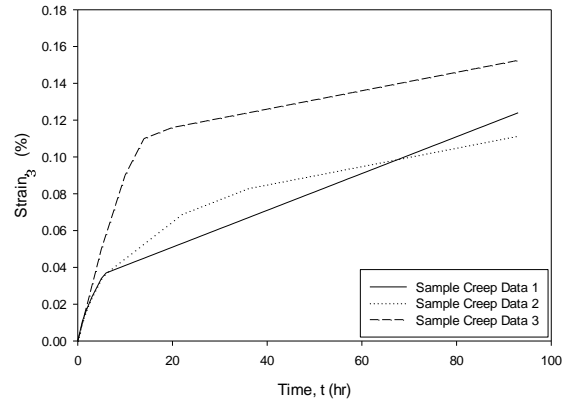


Figure 1: Examples of primary and secondary dominant creep.

CONSTANT DETERMINATION

Creep constants embedded in the frameworks of Eq. (1) or (3) are equivalently determined by a number of approaches; creep deformation history or creep deformation rate data from experiments are needed for each of the respective history and rate forms. Constants are typically developed by regression curve fitting against time first or with respect to stress. There are no techniques that have been established that help to analytically identify creep constants across various stress levels when data is sparse.

For the history formulation of Norton-Bailey Eq. (1), ideally the constants of the formation can be determined with a minimum of three creep points (termed 1, 2, and 3). The points, which take the form of ordered triples [i.e., (time, stress, strain)] could be derived from one curve at a given creep stress or points could be taken from separate experiments, such as the time to a given level of strain for creep curves at different stress level. The latter collection is preferred. Identical temperature is assumed. For either case, Eq. (1) is manipulated to develop a series of linear equations, i.e.,

$$\begin{aligned} \left(\log \frac{t_{r,2}}{t_{r,1}} \right) m + \left(\log \frac{\sigma_2}{\sigma_1} \right) n &= \log \frac{\epsilon_{cr,2}}{\epsilon_{cr,1}} \\ \left(\log \frac{t_{r,3}}{t_{r,2}} \right) m + \left(\log \frac{\sigma_3}{\sigma_2} \right) n &= \log \frac{\epsilon_{cr,3}}{\epsilon_{cr,2}} \end{aligned} \quad (5)$$

Without loss of generality, point pairs 1-2 and 2-3 are used; however, other combinations can be used equivalently. One restriction placed on Eq. (5) is that at least one strain level in the collection of points must be unique. Similarly, at least one stress level and one time coordinate in the collection must be unique. Otherwise, the coefficient matrix of the system becomes singular or the lines become non-intersecting. Constants m and n are derived from this linear system and used to develop an approximation of the creep strain coefficient, e.g.

$$A = \frac{\epsilon_{cr,i}}{(\sigma_i)^n (t_i)^m} \quad (6)$$

where i corresponds to either point 1, 2, or 3. This approach is repeated for each temperature level for which data exists. A tacit assumption for this two-step approach to lead to valid material properties is that candidate points must be derived from the primary or secondary regime of the creep curves.

REGRESSION ANALYSIS

Experiments generally require measuring a dependent and independent variable. To simulate similar data, the relationship between the two variables needs to be precisely calculated. Regression analysis provides a method for which constants can be calculated that allow a given function to best-fit the data [4]. Experimental creep deformation and data extracted from standard creep experiments can be reduced in two ways. The first technique involves keeping the time increments constant and measuring the strain at each point across multiple stresses. The second technique involves measuring the time it takes to reach set increments of strain. Both of these techniques can be better understood in Fig. 2.

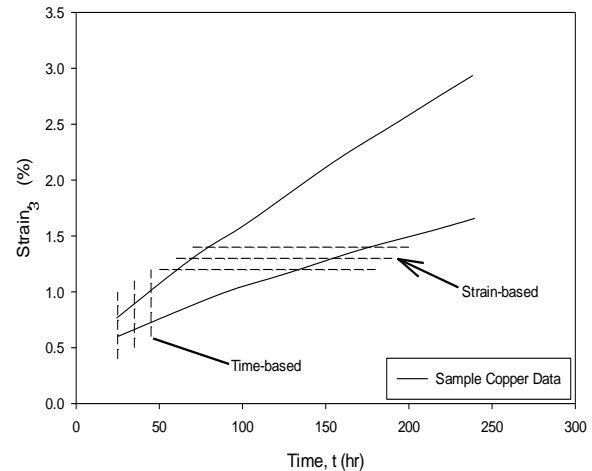


Figure 2: Creep data sampling methods

Since the Norton-Bailey is a *power law*, the equation for general power law regression fitting is used:

$$y = Bx^c \quad (7)$$

where B and c are found through the regression equations:

$$c = \frac{k \sum_{i=1}^k (\ln x_i \ln y_i) - \sum_{i=1}^k (\ln x_i) \sum_{i=1}^k (\ln y_i)}{k \sum_{i=1}^k (\ln x_i)^2 - \left(\sum_{i=1}^k \ln x_i \right)^2} \quad (8)$$

$$B = e^{\frac{\sum_{i=1}^k (\ln y_i) - c \sum_{i=1}^k (\ln x_i)}{k}} \quad (9)$$

Where k is the number of points used. Clearly, y can take the form of strain and x can stand for time or stress; however, this is only applicable for time-based data. Strain-based data could require x as strain and y as time or stress. The limitation of these equations when applying them to the Norton-Bailey power law is that it only models primary and secondary creep since tertiary creep is of an exponential nature. Also, x_i 's cannot be identical, as the denominator in Eq. (8) would become zero. Regression has also been applied to other curve-fitting techniques such as the Coffin-Manson equation [5].

BIVARIATE REGRESSION: TIME-BASED

The Norton-Bailey equation is distinct from Eq. (7) because of the t^m term. The regression approach of Eqs. (8) and (9) is plausible for this creep model since it is separable, i.e.

$$\varepsilon = f(T)g(\sigma)h(t) \quad (10)$$

For adequate power law regression, terms in the Norton-Bailey equation had to be factored together. For constant temperature, i.e. $f(T) = 1$, by grouping the left most terms in the first regression equation, the following results,

$$\varepsilon = (A\sigma^n)t^m \quad (11)$$

Comparing this equation to the general power law of Eq. (9) it is observed that

$$A\sigma^n = B \quad (12)$$

$$t^m = x^c \quad (13)$$

Thus, substituting the corresponding values: t for x and ε for y , into Eq. (8), the regression equation to determine m becomes:

$$m = \frac{k \sum_{i=1}^k (\ln t_i \ln \varepsilon_i) - \sum_{i=1}^k (\ln t_i) \sum_{i=1}^k (\ln \varepsilon_i)}{k \sum_{i=1}^k (\ln t_i)^2 - \left(\sum_{i=1}^k \ln t_i \right)^2} \quad (14)$$

To find n , the Norton-Bailey power law was rearranged into the form:

$$\varepsilon = (At^m)\sigma^n \quad (15)$$

where $At^m = B \quad (16)$

and $\sigma^n = x^c \quad (17)$

And after substituting σ for x and ε for y , into Eq. (8), the regression equation to determine n becomes:

$$n = \frac{k \sum_{i=1}^k (\ln \sigma_i \ln \varepsilon_i) - \sum_{i=1}^k (\ln \sigma_i) \sum_{i=1}^k (\ln \varepsilon_i)}{k \sum_{i=1}^k (\ln \sigma_i)^2 - \left(\sum_{i=1}^k \ln \sigma_i \right)^2} \quad (18)$$

Determination of the creep-strain coefficient, A , requires rearranging the Norton-Bailey equation into the form:

$$\varepsilon = A(\sigma^n t^m) \quad (19)$$

where A equals B and

$$(\sigma^n t^m)^1 = x^c \quad (20)$$

Inspecting Eq. (11) in this way requires using the previously found constants of n and m and then finding the coefficient, A , by substituting the corresponding values into Eq. (9).

$$A = e^{\frac{\sum_{i=1}^k (\ln \varepsilon_i) - (1) \sum_{i=1}^k \ln(\sigma^n t^m)_i}{k}} \quad (21)$$

Thus, the three coefficients in Eq. 1 can be calculated when time is the dependent variable.

BIVARIATE REGRESSION: STRAIN-BASED

This method involves solving the Norton-Bailey equation for time and reevaluating it to match the regression equation. The transformation of the Norton-Bailey equation takes on the form:

$$t = \left[\frac{\varepsilon}{(A\sigma^n)} \right]^{\frac{1}{m}} \quad (22)$$

Solving for m first requires the transformation of Eq. (22) into the form:

$$t = \left[\frac{1}{(A\sigma^n)^{\frac{1}{m}}} \right] (\varepsilon)^{\frac{1}{m}} \quad (23)$$

where $(A\sigma^n)^{\frac{1}{m}} = B \quad (24)$

and
$$\varepsilon^{\frac{1}{m}} = x^c \quad (25)$$

Thus, after equating the reciprocal of m with c and substituting in ε for x and t for y , the equation becomes:

$$m = \frac{k \sum_{i=1}^k (\ln \varepsilon_i)^2 - (\sum_{i=1}^k \ln \varepsilon_i)^2}{k \sum_{i=1}^k (\ln \varepsilon_i \ln t_i) - \sum_{i=1}^k (\ln \varepsilon_i) \sum_{i=1}^k (\ln t_i)} \quad (26)$$

To find n , Eq. (22) must be rearranged to:

$$t = \left[\frac{\varepsilon}{A} \right]^{\frac{1}{m}} (\sigma^{\frac{-n}{m}}) \quad (27)$$

where
$$\left[\frac{\varepsilon}{A} \right]^{\frac{1}{m}} = B \quad (28)$$

and
$$\sigma^{\frac{-n}{m}} = x^c \quad (29)$$

Equating Eq. (29) to Eq. (8), n is found to be:

$$n = -m \left[\frac{k \sum_{i=1}^k (\ln \sigma_i \ln t_i) - \sum_{i=1}^k (\ln \sigma_i) \sum_{i=1}^k (\ln t_i)}{k \sum_{i=1}^k (\ln \sigma_i)^2 - (\sum_{i=1}^k \ln \sigma_i)^2} \right] \quad (30)$$

Determination of the creep-strain coefficient, A , with this method, involves manipulating the equation into the form:

$$t = (A)^{\frac{-1}{m}} (\varepsilon^{\frac{1}{m}} \sigma^{\frac{-n}{m}}) \quad (31)$$

where
$$A^{\frac{-1}{m}} = B \quad (32)$$

and
$$(\varepsilon^{\frac{1}{m}} \sigma^{\frac{-n}{m}})^1 = x^c \quad (33)$$

After relating Eq. (32) to Eq. (9) and substituting in the corresponding values, A is found to be:

$$A = e^{-m \frac{\sum_{i=1}^k (\ln t_i) - (\sum_{i=1}^k \ln t_i) \frac{1}{k}}{\sum_{i=1}^k \ln(\varepsilon^{\frac{1}{m}} \sigma^{\frac{-n}{m}})_i}} \quad (34)$$

Thus, the three coefficients in Eq. 1 can be calculated when strain is the dependent variable.

SIMULATED RESPONSE

In order to validate the time-based approach, simulated data was constructed. This was done by assigning values to A , n , and m , and then choosing predetermined time and stress intervals as seen in Fig. 2. Then, strain values were calculated using those times and stresses with the constants by using the Norton-Bailey power law. In essence, this was an ideal data set for which the value of the constants can be reversed out using the regression method. To accomplish this, every factor in Eq. (14), Eq. (18), and Eq. (21), i.e. the natural logarithms of the times, stresses, and strains and the combinations between them, was calculated. In total there were 24 points, 12 for 150 MPa and 12 for 200 MPa, so the summation range was from 0 to 24. Using those equations and the calculated values, the constants were calculated and tabulated in Table 1. Substituting these values into the Norton-Bailey power law and graphing the resulting values netted creep curves that matched the constructed data perfectly, that is, with a correlation coefficient of 1.0, as expected since the data we regressed from was crafted from the Norton-Bailey equation, hence, Method 1 is proven to work. The results are shown in Fig. 3.

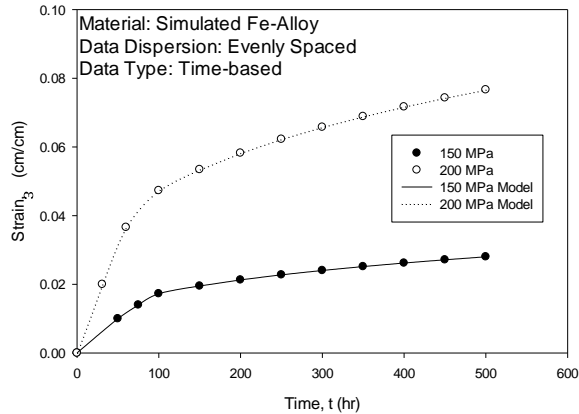


Figure 3: Comparison of simulated data and regression model for time-based conditions.

Table 1: Creep constants for first set of formulated data found using time-based method

A	$1.05(10)^{-10} \text{ MPa}^{-n} \text{ hr}^{-m} \%$
n	3.5
m	0.3

To prove a strain-based method, the constants were assigned different values, and intervals of stress and strain were chosen. The time values were calculated using that data and Eq. (23). This data can be found in Fig. 4, below.

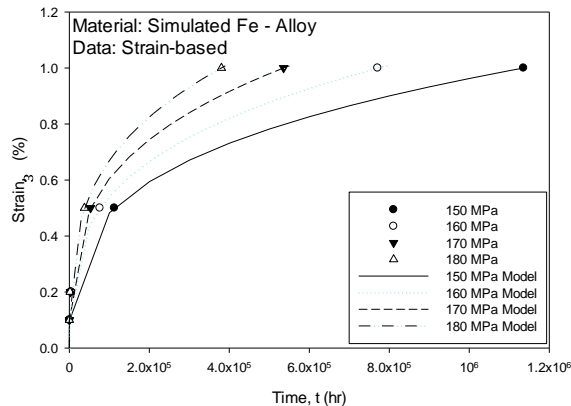


Figure 4: Comparison of simulated data and regression model for strain-based conditions.

All factors in Eq. (26), Eq. (30), and Eq. (34), are then calculated. For this data set, there were a total of 16 data points, meaning the summation range was from 0 to 16. Using those equations and the calculated values, the constants were calculated and tabulated in Table 2.

Table 2: Creep constants for second set of formulated data using strain-based method

A	$1.85(10)^{-6} \text{ MPa}^{-n} \text{ hr}^{-m} \%$
n	2.5
m	0.3

Substituting those constants along with the data into the Norton-Bailey power law resulted in curves that perfectly matched the formulated data, again proving the validity of using regression with the rearranged Norton-Bailey equation, Eq. (22).

ACTUAL RESPONSE

The model is applied to experimental data to see how strongly the models correlate to the actual data. The first set of data that is to be looked at is experimental creep data from copper [6]. This data is included in Fig. 5 along with the calculated constants used. Modeling with these constants, the correlation coefficient was 0.996 when modeling the 40 MPa data and 0.987 when modeling the 50 MPa data.

The next set of data was creep data from SUS316 stainless steel [7]. This data is included in Fig. 6 along with the model comparison and the constants found. When used in modeling, these constants resulted in a correlation coefficient of 0.998 when modeling 245 MPa and 0.988 when modeling 265 MPa.

Another material analyzed with the regression model was arc-cast tungsten [8]. The experimental data and the predicted model are shown in Fig. 7 with the constants found using the time-based regression method. The model had an R-squared value of 0.9983 when correlated against the 460 MPa data and 0.9956 when correlated against the 560 MPa data.

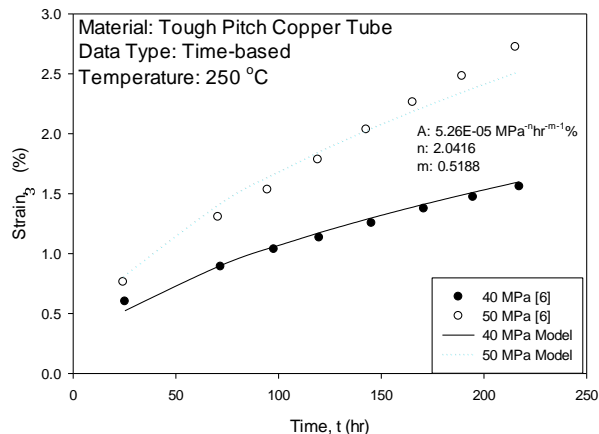


Figure 5: Comparison of experimental data and modeled creep data for tough pitch copper at 250 °C

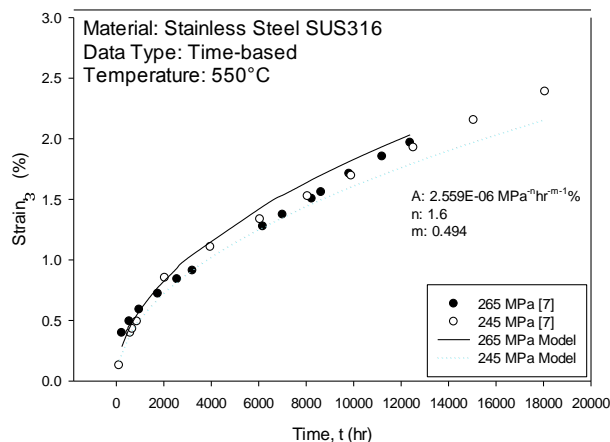


Figure 6: Comparison of experimental data and modeled creep data for SUS316 at 550 °C

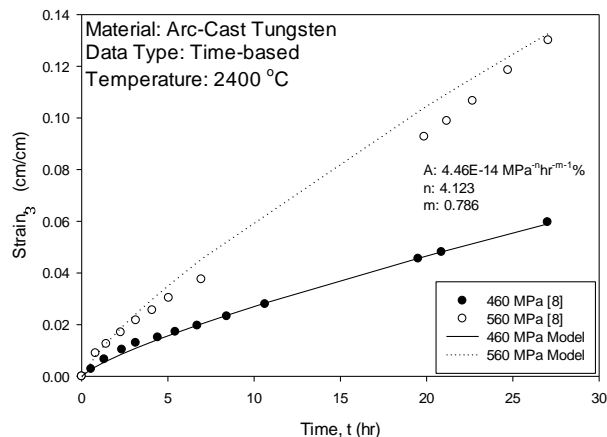


Figure 7: Comparison of experimental data and modeled creep data for arc cast tungsten at 2400 °C

BIVARIATE ANALYSIS

A notable characteristic of regression analysis is that it finds the best possible fit with

as few points as possible. The ideal data set has near continuously recorded data which would confer the most accurate constants. Either of these approaches can be applied with much less data that has some restrictions. Theoretically, constants can be determined with these methods with as little as 4 points. This is because the denominator of the equation for n and m becomes zero when only one x -value, i.e. time and stress, are used in method one. For method two, n and m become zero when one strain or stress is used. This means that, not only must there must be at least two different times and stresses or strains, but they must be the same two times and stresses or strains.

Applying this to physical data resulted in the following observations. Clearly, more data points result in a better fit. Determining the number of points to use depends on how accurate of a fit is needed and how much physical time is available to collect data e.g. multiple experiments over the course of months, or one experiment done in a day. The caveat of this approach is that the closer the time intervals used between each stress, the better the fit. In most cases, using vastly different times between the stresses resulted in negative constants and therefore no model could be constructed. Even in simulated data where the constants were simulated and the strains were formulated with the Norton-Bailey power law, using different times across each stress, even by very small amounts, resulted in less than ideal constants. Using this method on data extracted from graphs will be less accurate than analyzing the exact data itself.

CONCLUSION

Limited experimental data on materials used in pressure turbines makes accurate creep prediction difficult. The focus of this investigation is to determine a method for determining temperature-dependent creep constants for modeling creep fatigue when there is limited data. Two methods were developed and validated: one for use when the data assumes that strain is the independent variable and the other when the data assumes that time is the independent variable. The limitations of these techniques were also analyzed, showing that while this method could be used on as little as four points that if accuracy was desired that as many points as possible should be used. Use of these methods on available creep data showed

accurate prediction and will be used on future experimental data.

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